Finding the 'n' dB point for a first order filter

This analysis is useful when one wishes to set the power bandwidth of an amplifier using two first order networks that act essentially independently. Only the high pass case is considered however the derivation can be applied to the low pass case.

For the High Pass Network:

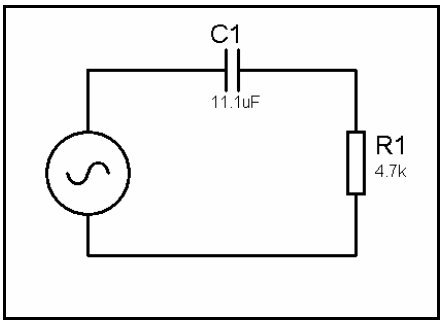


Figure 1: High Pass First Order Network

Find the current in C1 and R1.

$$I = \frac{V_{IN}}{Z_{TOTAL}} = \frac{V_{IN}}{R - \frac{j}{\omega C}}$$

Multiply numerator and denominator by complex conjugate of denominator (removes imaginary term from denominator)

$$I = \frac{V_{IN}\left(R + \frac{j}{\omega C}\right)}{\left(R - \frac{j}{\omega C}\right)\left(R + \frac{j}{\omega C}\right)} = \frac{V_{IN}\left(R + \frac{j}{\omega C}\right)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}$$

Output voltage is developed across R1:

$$V_{OUT} = I \times R = \left[\frac{V_{IN}\left(R + \frac{j}{\omega C}\right)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}\right] \times R$$

Assuming that only the magnitudes are of interest:

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$$V_{OUT} = \left(V_{OUT} \times V_{OUT}^{*}\right)^{1/2}$$

$$V_{OUT} = \left(\left[\frac{R \times V_{IN}\left(R + \frac{j}{\omega C}\right)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}\right] \times \left[\frac{R \times V_{IN}\left(R - \frac{j}{\omega C}\right)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}\right]\right)^{1/2}$$

(V*out is the complex conjugate of Vout)

$$V_{OUT} = \left(\frac{{V_{IN}}^2 R^2 \left(R^2 - \left(\frac{j}{\omega C}\right)^2\right)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)^2}\right)^{1/2}$$

The common terms from numerator and denominator cancel leaving:

$$V_{OUT} = \frac{V_{IN} \times R}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)^{1/2}}$$

Note: This result is the same as that found from treating the network as a potential divider and finding the absolute value of the denominator…

$$V_{OUT} = \frac{V_{IN} \times R}{R + \frac{1}{j\omega C}} = \frac{V_{IN} \times R}{R - \frac{j}{\omega C}}$$

The above is legal because 1/j = -j…

$$V_{OUT} = \frac{V_{IN} \times R}{\left(R^2 - \left(\frac{j}{\omega C}\right)^2\right)^{1/2}}$$

Don’t forget that

$$j^2 = -1$$

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Which gives

$$V_{OUT} = \frac{V_{IN} \times R}{\left(R^2 + \left(\frac{1}{\omega C} \right)^2 \right)^{1/2}}$$

To find the n dB point:

$$\frac{V_{OUT}}{V_{IN}} = G$$

$$G = \frac{R}{\left(R^2 + \left(\frac{1}{\omega C} \right)^2 \right)^{1/2}}$$

Multiplying by the denominator

$$G\left(\left(R^2 + \left(\frac{1}{\omega C} \right)^2 \right)^{1/2} \right) = R$$

Squaring all terms

$$G^2\left(R^2 + \left(\frac{1}{\omega C} \right)^2 \right) = R^2$$

Multiplying out the outer brackets

$$G^2R^2 + G^2\left(\frac{1}{\omega C} \right)^2 = R^2$$

Subtracting the term in G & R from both sides

$$G^2\left(\frac{1}{\omega C} \right)^2 = R^2 - G^2R^2$$

Dividing by G^2

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$$\left(\frac{1}{\omega C}\right)^2 = \frac{R^2 - G^2R^2}{G^2}$$

Taking the reciprocal of both sides

$$(\omega C)^2 = \frac{G^2}{R^2 - G^2R^2}$$

Dividing by C and taking the root of both sides

$$C = \left(\frac{G^2}{\omega^2 R^2 (1 - G^2)}\right)$$

It is more common to design around C and choose R to be the subject of the above equation. However in the topology discussed in the “ice ‘n’ lemon” preamplifier R is used to define the gain of an amplifying stage and therefore is already fixed.

Taking some values R = 4.7k G = 0.9885 (-0.1 dB) and ω = 125.66 (f = 20Hz; ω=2πf)

$$C = \left(\frac{0.9885^2}{125.66^2 \times (4.7 \times 10^3)^2 \times (1 - 0.9885^2)}\right)^{1/2}$$

$$C = 11.1\mu F$$

The process can be repeated for a low pass network remembering that the reactance of C is now the component that the output voltage is developed across.

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